# Effect of traffic rule breaking behavior on pedestrian counterflow in a channel with a partition line

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In this paper a partition line is used in the counterflow system to present the default (conventional) traffic rule: pedestrians prefer to walk on a certain side on the road during movement, e.g., the right-hand side in China or the left-hand side in Japan. Based on the counterflow model of Takimoto (model A), we introduced two modified models, i.e., model B and C, to study the effects of a partition line in the consideration of people who do not obey the default traffic rule. Model B represents that factor in time scale, while model C in space scale. In model B, there are pedestrians who cross the partition line but choose not to obey the default traffic rule with a probability  $p_{nor}$ , while in model C, if a pedestrian crosses the partition line and goes away from it further than a certain nonobeying-rule threshold distance  $d_t$ , he will not obey the traffic rule. It is found that the behavior of traffic rule breaking influences much the counterflow when it is at the choking flow state rather than at the free moving or stopped state. Furthermore, it is shown that the default traffic rule is not always positive to the counterflow in all situations. It depends on the game result of these two opposite sides: to use the channel width as much as possible and to avoid the interference from the other group as far as possible.

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## I. INTRODUCTION

Pedestrian flow is a complex system composed of many particles with various interactions. Recently, it has attracted considerable attention [1] and has been frequently researched in many different ways such as the social force model [2] and cellular automata (CA) models [3]. Helbing *et al.* [4] studied crowd flow escaping from a room and collective phenomena such as arching, fast is slower were reproduced. Tajima and Nagatani [5] investigated crowd flow going outside a hall and found that the mean flow rate and transition time scale by the exit size in the choking-flow region. Tajima et al. [6] also investigated pedestrian channel flow through a bottleneck under the open boundaries. It is shown that in the choking-flow region, the saturated flow rate scales by the bottleneck width, while the critical density scales by the ratio of the bottleneck width to the channel width. Inspired from the phenomenon of chemotaxis in biology, Burstedde et al. introduced the floor field model [7,8] using static floor field and dynamic floor field to translate a long-ranged spatial interaction into an attractive local interaction, but with memory. This kind of model is one of the most reliable CA models that have been tested experimentally and theoretically by many researchers [9-11].

As a typical pedestrian flow phenomenon, counterflow in different situations has also been studied extensively. Helbing and co-workers [2,12] studied counterflow in a straight path using the social force model and reproduced lane formation. The scenario of two groups heading in the opposite directions through a narrow door was also simulated. In the early stages of counterflow research using CA models, a simple update mechanic [13–15] was used: the two groups involved move in turn with odd time steps and even time

steps. Biham and Middleton [13] studied the jamming tran-



FIG. 1. Schematic illustration of the pedestrian counterflow in a channel with the partition line of model A. The top and bottom sides of the channel are walls. The left and right sides are open boundaries. The partition line is positioned on the center of the channel, which is indicated by the dotted line. The right (left) walker going to the right (left) is indicated by the full (open) circle, and comes into the channel from the upper (down) half of the left (right) boundary with a constant entrance density  $p_l(p_r)$ . Arrows indicate possible moving directions of the two groups. The arrows with the bold border indicate the corresponding particles are under control of the default traffic rule.

sition on a square lattice with two kinds of particles: the one moves from left to right and the other moves from down to up. All the particles can only move in a forward direction or stay. Although it was the traffic flow they modeled, the model rules are so simple that the model can be seen as a prototypical model for pedestrian flow. It is shown that the sharp first-order jamming transition is due to the excluded volume effect. Fukui and Ishibashi [14] studied the counterflow that consists of one right walker and many left walkers in a passage. Pedestrians in their model can move sideways. Thereafter, they [15] checked the counterflow in its general sense which consists of two groups of walkers heading in opposite directions.

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FIG. 2. Schematic illustration of model B. The dotted arrows with the bold border indicate the corresponding particles are under control of the default traffic rule with probability  $P_{nor}$ .

Muramatsu *et al.* [16] mimicked the pedestrian counterflow in a channel with open boundaries using a biased random walker model. Heading drift is introduced to express the desire of movement in the heading direction and this time all the walkers within the channel are updated once at each time step. Muramatsu and Nagatani [17,18] investigated jamming transition in the counterflow composed of four groups at a crossing and compared the results with those of two-way counterflow.

Many CA models considering more parameters were introduced afterwards. Tajima *et al.* [19] extended the CA model by taking into account following the front persons with the same direction and avoiding the front persons with the opposite direction. Pattern formation and jamming transition in these two CA variants were simulated and compared. Blue *et al.* [20] took into consideration the positionexchange factor in three kinds of bidirectional pedestrian flow: flows in directionally separated lanes, interspersed





FIG. 3. Schematic illustration of model C. Only pedestrians who have crossed the partition line and are within the range of  $d_t$  are under control of the default traffic rule.

flow, and dynamic multilane flow. Li *et al.* [21] considered position-exchange and back-stepping factors. Yu and Song [22] modeled the pedestrian counterflow in a channel considering the surrounding environment. Itoh and Nagatani [23] simulated shifting of the audience between two halls. It is found that there exists an optimal admission time for shifting the audience. The counterflow in different scenarios such as in a T-shape channel [24], with different velocities [25], in places where people are going on all fours [26,27] have also been investigated.

Takimoto *et al.* [28] investigated the effect of the partition line on the pedestrian counterflow. A longitudinal drift was introduced to present the pedestrians' desire to return to the preferable side once they have been crowded into the other side of the partition line. It is shown that the partition line has a significant effect on the counterflow and enhances the critical current and density.

In this paper, we use Takimoto's model as a raw model to study whether the jamming transition in a channel with a

FIG. 4. (Color online) Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against the left boundary density  $p_l$  for  $p_r=0.1, 0.2, \ldots, 0.9$  under different system sizes in model A.



FIG. 5. (Color online) Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against the probability of not obeying rule  $P_{nor}$  for various values of  $p_r$  and  $p_l$  where L=100, W=40,  $D_1=0.6$ , and  $D_2=0.92$  in model B.

partition line depends on system size or not. And then two variants of the raw model are presented to investigate what would happen if all pedestrians do not obey the default traffic rule.

## **II. MODEL**

The raw model, hereafter called model A (Fig. 1), is the same as the model in Ref. [28]. We also use this model to simulate the counterflow in a channel under open boundaries. There are two types of walkers with no back step in this system: the right walker going from the left to the right and the left walker going from the right to the left. The right (left) walker prefers to move within (outside) the partition line. When the right (left) walkers are crowded outside (inside) the partition line, the hopping probabilities matrix is calculated differently because of the default traffic rule. That traffic rule means the convention of sideways direction preference during movement in a society. In model A, the conventional traffic rule is represented by a tangential drift  $D_2$ , while the original heading drift, which is also called longitudinal drift here, accordingly is denoted by  $D_1$ . The possible configurations of a pedestrian and the transition probabilities scheme corresponding to each configuration are given in detail in Ref. [28]. For consistence, and the reason that there is



FIG. 6. (Color online) Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against  $p_l$  for various values of  $p_r$  and  $P_{nor}$  where L = 100, W = 40,  $D_1 = 0.6$ , and  $D_2 = 0.92$  in model B.

actually no intrinsic difference between left-hand side preference and right-hand side preference, we use the same arrangement of walkers as Ref. [28]. The right (left) walkers enter the channel from the up (down) half of the left (right) boundary, move along the channel, and come out of the right (left) boundary.

Based on model A, we proposed models B and C. Model B (Fig. 2) is a variant of model A. All the walkers who cross the partition line will obey the traffic rule and are under the control of  $D_2$  in model A. But in model B there is a question of probability. It means that there are pedestrians who cross the partition line but choose not to obey the conventional traffic rule. The probability of not obeying the rule is denoted as  $P_{nor}$ . It reflects the fact that not all pedestrians in a crowd observe the traffic rule. This deviation can be used to check the robustness of model A.

Model C (Fig. 3) is another variant. If a pedestrian crosses the partition line and goes away from it further than a certain not-obeying-rule threshold distance  $d_t$ , he will not obey the traffic rule. It represents the fact that people do not return to their original region on the other side if they have gone too far from the partition line in a real crowd to avoid the potential strong interference from pedestrians of the other group.



FIG. 7. The typical patterns obtained using model B where L=100, W=40,  $D_1=0.6$ , and  $D_2=0.92$ . The pattern (a) shows the freely moving phase obtained at  $p_r=p_l=P_{nor}=0.3$ . (b)  $p_r=p_l=0.3$  and  $P_{nor}=0.8$ . The patterns (c)–(f) show the time evolution of the jamming transition for  $p_r=p_l=P_{nor}=0.4$ : (c) the pattern obtained at t=959, (d) the pattern obtained at t=1104, (e) the pattern obtained at t=4830, and (f) the pattern obtained at t=5014.

## **III. SIMULATION AND RESULTS**

We have carried out simulations on the counterflow in a  $W \times L$  channel without back steps. The entrance density on the left (right) boundary  $p_l(p_r)$  has been kept constant during each simulation. For different  $p_l$  and  $p_r$  combinations, the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  have been checked. The mean velocity  $\langle v \rangle$  at one update step is defined as the number of walkers who choose to move forward divided by the total number of walkers in the channel. The occupancy  $\rho$  is defined as the total number of walkers in the channel divided by the channel area  $(W \times L)$ .

For each simulation, 10 000 time steps have been carried out, and the values of  $\langle v \rangle$  and  $\rho$  were computed according to the last 4000 time steps averaged over 20 runs.

First we give the simulation results of model A (Fig. 4) When the right entrance density  $p_r$  is small, the increase of the left entrance density  $p_l$  does not cause blockage, but does result in the decrease of mean velocity. This is because there are more pedestrians in the channel but less free cells for them to move into. However, when  $p_r$  is larger, the jamming transition appears with the increase of  $p_l$ . It is also shown that the critical left entrance density  $p_{lc}$  at the transition point is independent of the system size.

The simulation results of model B are given in Figs. 5–7. Figure 5 shows the plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against the probability of not obeying rule  $P_{nor}$  for various values of  $p_r$  and  $p_l$ , where L=100, W=40,  $D_1$ =0.6, and  $D_2$ =0.92 using model B. When  $p_1$  and  $p_r$  are small, there are not many pedestrians in the channel, therefore only a few pedestrians walk across the partition line and  $P_{nor}$  does not has much influence on the crowd flow. As shown in Fig. 5, e.g., when  $p_l = p_r = 0.1$ , the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  remains at almost a constant value with the increase of  $P_{nor}$ . However, when  $p_l = p_r = 0.4$ , the number of total pedestrians in the system becomes bigger, the interaction among the crowd becomes stronger, and thus more pedestrians are pushed across the partition line. The effect of the probability of not obeying traffic rule rises. The larger  $P_{nor}$  is, the more pedestrians who have crossed the partition line do not desire to return to their original side and choose to stay in the opposite side to face the interactions from pedestrians of the other group. This causes an inefficient movement. It is found that the default traffic rule is positive to enhance the crowd flow in this case.

But is the traffic rule always useful to an efficient movement? To make things clear, we plot the mean velocity  $\langle v \rangle$ and the occupancy  $\rho$  against  $p_l$  for various values of  $p_r$  and  $P_{nor}$  in Fig. 6. The dotted line indicates the results for  $P_{nor}$ =0, which means we are not considering the effect of  $P_{nor}$ . As shown in Fig. 6, if  $p_r < 0.3$ , taking into account  $P_{nor}$  does not influence the crowd flow when  $p_l$  is also small. But when  $p_l$  becomes bigger than 0.6, considering  $P_{nor}$  can improve the movement. That is to say, when the two entrance densities



FIG. 8. (Color online) Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against  $p_l$  without a partition line where  $p_r=0.1$ , L = 100, W=40,  $D_1=0.6$ , and  $D_2=0.1$ .

are too much asymmetrical, it can be helpful for the crowd flow if some of those people who have crossed the partition line choose not to return to the original side in a hurry. Take  $p_r=0.2$ ,  $p_l=0.7$  as an example, the results of which are also plotted in Fig. 5, right walkers within the partition line outnumber left walkers outside the line. So there are more right walkers than left walkers who have crossed the line. For those right walkers outside the line, it is better to stay in this scarcely populated area rather than to return to the original crowded side. Although it is a place on which left walkers are distributed, it still is an acceptable strategy when the density of the left walkers is small and thus the interactions are not strong. However, if  $p_r$  and  $p_l$  are both kind of large, e.g.,  $p_r = p_l = 0.4$ , trying not to obey the conventional traffic rule is a bad idea, and results in congestion quickly. It should be noticed that both the two condition combinations  $(p_r = p_l)$ =0.4 and  $p_l$ =0.7 and  $p_r$ =0.2) are at the choking flow stage of the crowd flow. As we can see in Fig. 6,  $P_{nor}$  does almost nothing about the crowd flow when the  $p_r$  and  $p_l$  combination is at the free moving state or at the perfect stop state. It has a great influence on the crowd flow if the condition combination falls in the range where the phase transition hap-



FIG. 9. (Color online) Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against  $d_t$  for various values of  $p_r$  and  $p_l$  where L = 100, W = 40,  $D_1 = 0.6$ , and  $D_2 = 0.92$  in model C.

pens. Therefore, if the phase transition is sharp, the traffic flow is robust when some people do not move by the rules, but if the phase transition is smooth, the traffic flow will be susceptible to  $P_{nor}$ . How it is affected depends on the values of  $p_r$  and  $p_l$ , i.e., whether it is the case of both large values such as  $p_r=p_l=0.4$ , or it is the case of a very asymmetrical combination such as  $p_l=0.7$  and  $p_r=0.2$ .

We study the flow patterns at the free moving stage and observe the jamming transition evolution. Figure 7 shows the typical patterns obtained using model B where L=100, W =40, D1=0.6, and  $D_2=0.92$ . The pattern 7(a) shows the free moving phase at  $p_r = p_l = P_{nor} = 0.3$  and 7(b) for  $p_r = p_l = 0.3$ and  $P_{nor}=0.8$ . There is not much difference between Figs. 7(a) and 7(b). It shows  $P_{nor}$  has little effect on the flow patterns when the entrance densities are small. Also, we can see in patterns 7(a) and 7(b) that there are a few pedestrians who walk deep into the opposite region. This is because  $P_{nor}$ allows some people not to obey the default traffic rule. The patterns 7(c)-7(f) show the time evolution of the jamming transition for  $p_r = p_l = P_{nor} = 0.4$ : Figure 7(c) shows that a small cluster of competing right and left walkers arises near the upper-left entrance and may cause an early jam. In Fig. 7(d) the cluster formed before is dismissed. In Fig. 7(e) a new larger cluster appears at the bottom-right part of the channel and it seems to be forming a jam. In Fig. 7(f) the cluster grows bigger and bigger and finally causes a perfect blockage. It is observed that the clusters often appear at a



FIG. 10. Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against  $p_l$  for various values of  $p_r$  and  $d_t$  where L=100, W=40,  $D_1=0.6$ , and  $D_2=0.92$  in model C.

region inside the crowd, as opposed to a region near the partition line as shown in Ref. [28].

We have also checked the scenario when the partition line is not in the center but at the fraction of 7/9 of the channel width, as a comparison with the case  $p_l=0.7$  and  $p_r=0.2$ . It is shown that with the same in flow of pedestrians and the ratio of the total number of left walkers to that of right walkers, it is better to keep each group on its own side and to put the partition line in the center so that each group can move more freely. If the partition line is much down to the bottom edge (or up to the upper edge, it is the same thing), there is little room for the right walkers and a slight turbulence can result in a severe jam easily. The results are consistent with what we have found in Ref. [22].

What would happen if there is no partition line at all (which means pedestrians can enter into the channel from the whole boundaries), but all the pedestrians are still under the control of the lateral drift regardless of their positions? Figure 8 shows the plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against  $p_l$  where  $p_r=0.1$ , L=100, W=40,  $D_1=0.6$ , and  $D_2=0.1$ . Comparison with the results of  $P_{nor}=0$  and  $P_{nor}=0.1$  shows that without the partition line, the two groups, i.e., left walkers and right walkers, have a large interference front edge (i.e., the whole width of the channel). And the strong interaction results in a traffic jam easily even



FIG. 11. (Color online) Plots of the mean velocity  $\langle v \rangle$  and the occupancy  $\rho$  against breaking rule degree  $D_b$  for various values of  $p_r$  and  $p_l$  where L=100, W=40,  $D_1=0.6$ , and  $D_2=0.92$  as a comparison of models B and C.

when  $p_l$  and  $p_r$  are small. While with the same  $p_l$  and  $p_r$  combination, considering the partition line and considering  $P_{nor}$  are good for efficient movement. Even in the free moving stage, without the partition line is still the worst strategy among these three situations because it has the most large interference front edge, more disordered movement choice, and thus less usage of the channel width. This is also the reason why partition lines are common in our daily life and why the conventional traffic rules have arisen.

Now let us investigate the simulation results using model C (see Fig. 9 and Fig. 10) where L=100, W=40,  $D_1=0.6$ , and  $D_2=0.92$ . The dotted line indicates results for  $d_t=20$ , which means not considering the effect of  $d_t$ . It is also found that  $d_t$  has little influence on the crowd flow at the free moving state or at the perfect stop state, while it greatly influences at the choking flow state. The difference between the two condition combinations of  $p_r$  and  $p_l$  mentioned before is also distinguished. It is observed that for the  $p_1=0.7$  and  $p_r$ =0.2 combination, the not-obeying-rule threshold distance  $d_t$ plays a more important rule when it falls within the range [3,10]. In this range, the larger  $d_t$  is, which means the more probably pedestrians choose to obey the traffic rule, the lower the mean velocity is. So the traffic rule is harmful to the crowd flow in this case. For all positive  $d_t$  values, obeying the traffic rule is either useless or harmful to the movement process. This is sort of contrary to what we assumed that the larger value of  $d_t$  may be useful to the pedestrian crowd flow.

Since  $P_{nor}$  and  $d_t$  are both parameters describing the degree of breaking the default traffic rule, we plot them together in Fig. 11 and use  $D_b$  to denote that degree to simplify the following explanation. For model B,  $D_b = P_{nor}$ . For model C,  $d_t$  is transformed to  $D_b$  through the formula  $D_b = (20$  $-d_t$ /20, where 20 is half of the channel width W=40. As shown in Fig. 11, for both two parameters combinations of  $p_r$ and  $p_l$  distinguished before, the interaction between  $D_h$  and crowd flow is severe and mutable at the early stage  $D_b$  $\in [0, 0.2]$  in model B. While in model C, this happens at the rear region  $D_b \in [0.6, 0.8]$ , corresponding to small values of  $d_t$ . It should be mentioned that the results in models B and C at  $D_{b}=0$  for  $p_{r}=p_{l}=0.4$  is kind of different. This is because the crowd flow is in the phase transition region when  $p_r$  $=p_1=0.4$  (Fig. 10). Jamming transitions take place with the probability depending on the random initial patterns. However, that fluctuate is not considered significant when we focus on the trends of curves on the whole  $D_b$  definition field. The trend of  $\langle v \rangle$  or  $\rho$  according to  $D_b$  is significant different and almost opposite at large for the two condition combinations  $(p_r = p_l = 0.4, p_l = 0.7, \text{ and } p_r = 0.2)$  in both models B and C.

### **IV. CONCLUSIONS**

In this paper the partition line is used to investigate the counterflow in a straight channel. People usually prefer to walk on a certain side of the partition line during movement, e.g., the right-hand side in China or the left-hand side in Japan. First we analyze a simple cellular automation model PHYSICAL REVIEW E 76, 026102 (2007)

(model A) introduced by Takimoto et al. and observed that the phase transition is independent on the system size. Then two variants of model A, model B and model C, are introduced to study how people who do not move by the rules influence the crowd flow. Model B uses the nonobeying-rule probability  $P_{nor}$  to represent the behavior of not obeying the traffic rule, and model C uses the nonobeying-rule threshold distance  $d_t$  to represent the behavior of breaking the traffic rule. In other words, model B reflects the rule breaking behavior in time scale, while model C reflects that behavior in space scale. It is shown that the rule breaking behavior plays a key role in the stage where the phase transition takes place rather than in the free moving stage or stopped stage. In the phase transition stage, two types of combinations of  $p_r$  and  $p_l$ are distinguished. The comparison between them indicates the conventional traffic rule is not always positive to the counterflow. For the case with asymmetrical entrance densities such as  $p_r=0.2$  and  $p_l=0.7$ , not obeying the traffic rule means the total width of the channel can be taken full advantage of. While for the case with symmetrical and larger entrance densities such as  $p_r = p_l = 0.4$ , breaking the traffic rule causes more severe interaction within the two groups of left walkers and right walkers. To use the channel width as much as possible is in direct contradiction with avoiding the interference from the other group as far as possible. The game result of these two opposite sides decides whether the traffic rule is positive to the counterflow.

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